Nonlinear conductivity and composition of partially ionized plasmas in a strong electric field

K. Morawetz
Max-Planck-Gesellschaft, Arbeitsgruppe "Theoretische Vielteilchenphysik," Universität Rostock, 18055 Rostock, Germany

M. Schlanges
Fachbereich Physik, Abteilung Theoretische Physik, Ernst-Moritz Arndt Universität Greifswald, 17489 Greifswald, Germany

D. Kremp
Fachbereich Physik, Universität Rostock, 18055 Rostock, Germany
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A partially ionized hydrogen plasma is considered, under the influence of a strong electric field. The nonlinear electrical conductivity is calculated in a self-consistent manner from the quantum transport cross sections of the electron-scattering processes and from a field-dependent nonequilibrium mass-action law in order to determine the plasma composition. Field effects dominate the electric transport properties at low densities because of the high ionization rates. At high densities, a strong increase of the conductivity describes the Mott effect caused by the many-body effects in the dense plasma. A minimum of the degree of ionization and the conductivity assign the interplay between field and many-particle effects. This minimum is reinforced by the field dependence of the mass-action law.

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I. INTRODUCTION

The transport properties of dense partially ionized plasmas are of special importance because of their relevance for the investigation of particle-beam–plasma interactions, for astrophysical problems, and for the development of high-power light sources [1–3]. An important quantity to describe the plasma properties is the electrical conductivity. There are many papers dealing with the calculation of this transport coefficient [4–11]. In most of the papers the linear-response regime is assumed valid for applied weak electric fields. Under such conditions the electrical conductivity is a field-independent quantity and it can be calculated solving the kinetic equation for small deviations from thermodynamic equilibrium. The collision processes may be considered, neglecting field effects, and the plasma composition is determined by a Saha equation, valid for the thermodynamic equilibrium.

In recent years there has been a growing interest concerning the investigation of the nonequilibrium properties of dense partially ionized plasmas in strong electric fields. In this case, the assumptions made above for weak fields cannot be applied. Introducing an electrical conductivity again, the latter now shows a nonlinear field dependence determined by the kinetic equation valid for arbitrary field strength. The following field effects must be taken into account: (i) the explicit nonlinear field dependence of the electron distribution function; (ii) the collision integrals are influenced by the strong electric field; and (iii) the plasma composition has to be determined from a nonequilibrium field-dependent mass-action law. A method used in order to solve the kinetic equation, also used in the paper presented here, is to expand the electron distribution function in terms of the anisotropy caused by the electrical field [12–14]. In the diffusion approximation of the collision integrals, the isotropic part of the distribution function is given by a Davydov-type expression. The conductivity is expressed then by the field-dependent distribution function, the scattering cross sections, and the number densities of the different plasma species. As already mentioned, the number densities cannot be calculated from Saha equations of thermodynamic equilibrium, as it may be done in the linear-response regime. The applied strong electric field gives rise to considerable changes in the plasma composition in comparison to that of thermodynamic equilibrium. Therefore, the field-dependent plasma composition as well as the field-dependent electrical conductivity have to be determined in a self-consistent manner from the kinetic equation. The number densities now follow from nonequilibrium rate equations instead of Saha equations [14]. That requires the knowledge of the field-dependent ionization and recombination rate coefficients determined by quantum statistical expressions given by the considered kinetic equation [15, 16].

The paper is organized as follows. In Sec. II the kinetic equation is discussed, which is used to describe the nonequilibrium properties of a partially ionized hydrogen plasma. The solution of the kinetic equation then is presented in Sec. III, applying the diffusion approximation and taking into account three-body collisions such as elastic, excitation, and ionization processes. In Sec. IV the field-dependent plasma composition then is calculated from a nonequilibrium field-dependent mass-action law. Finally, the results for the electrical conductivity are presented in Sec. V, including the relaxation effect in a first approximation.

II. KINETIC EQUATION FOR NONIDEAL PLASMAS

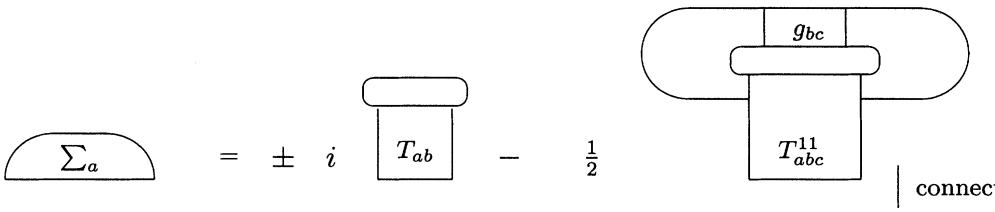
In order to describe the transport properties of dense partially ionized plasmas one has to start from quantum kinetic equations which take into account the formation and the decay of bound states as well as many-body effects. Such equations were given in three-particle collision approximation using the method of nonequilibrium real-time Green's functions [17, 18]. For the Wigner distribution function f_a of the free particles of species a in an external field $U(R,t)$, this equation can be written in the form

$$\left(\frac{\partial}{\partial t}+\frac{\partial E_a(p,Rt)}{\partial p}\frac{\partial}{\partial R}-\frac{\partial E_a(p,Rt)}{\partial R}\frac{\partial}{\partial p}\right)f_a(p,Rt)=\sum_b I^B_{ab}(p,Rt)+\sum_{b,c}I_{abc}(p,Rt)+\sum_b\frac{\partial}{\partial t}\epsilon\frac{\partial}{\partial\epsilon}\frac{1}{\epsilon}I^B_{ab}(p,Rt). \tag{2.1}$$

On the right-hand side the two- and three-particle scattering processes are taken into account by the corresponding collision integrals discussed below. On the left-hand side the drift of quasiparticles is described with energies following from the dispersion relation

$$E_a(pRt)=\frac{p^2}{2m_a}+Re\Sigma^R_a(p\omega Rt)|_{\hbar\omega=E_a(pRt)}+U(Rt). \tag{2.2}$$

The real part of the retarded self-energy function Σ^R_a gives the shift to the energy of an isolated particle due to the influence of the surrounding plasma particles. In the considered three-particle collision approximation we have in diagrammatic representation



(2.3)

where Σ_a denotes the causal self-energy function.

The two-particle T matrix is determined by the generalized Lippmann-Schwinger equation

$$T_{ab}(z)=V_{ab}+V_{ab}\frac{(1-f_a-f_b)}{z-H^{\rm eff}_{ab}}V_{ab} \tag{2.4}$$

with $H^{\rm eff}_{ab}=E_a+E_b+(1-f_a-f_b)V_{ab}$ being an effective Hamiltonian for the two-body problem in the medium. The three-particle processes are described by the T operators

$$T^{\kappa\kappa'}_{abc}=V^{\kappa}_{abc}+i^2V^{\kappa}_{abc}\frac{N_{abc}}{z-H^{0\rm eff}_{abc}-N_{abc}V^0_{abc}}V^{\kappa'}_{abc}, \tag{2.5}$$

which determine the transition probabilities between the initial and the final three-body scattering channels (κ is the channel quantum number). $H^{0\rm eff}_{abc}$ is the Hamiltonian of three free quasiparticles with single-particle energies given by (2.2). Furthermore, V^{κ}_{abc} denotes the potential in the channel κ and N_{abc} is the phase-space occupation factor.

To work with the energy shifts given by (2.2) is a difficult problem. But it is sufficient in many cases to approximate $E(pRt)$ by a momentum-independent quasiparticle shift [19]. Then one has instead of (2.2)

$$E_a(pRt)=\frac{p^2}{2m_a}+\Delta_a(Rt)+U(Rt).$$

In this approximation, the shift is related to the chemical potential according to

$$\mu_a=\mu^{\rm ideal}_a+\Delta_a,\quad \text{i.e.},\quad \Delta_a=\mu^{\rm corr}_a\ . \tag{2.6}$$

On the right-hand side of the kinetic equation (2.1) the collision integrals of the two- and three-particle scattering processes are taken into account. I^B_{ab} is the quantum-mechanical Boltzmann collision term

TABLE I. The notation of the multichannels used in 2.8.

κ	channel	$\lvert\kappa\alpha\rangle$	E_κ	f_κ	N_κ
0	$a+b+c$	$\lvert p_a\rangle\lvert p_b\rangle\lvert p_c\rangle$	$E_a+E_b+E_c$	$f_af_bf_c$	$(1-f_a)(1-f_b)(1-f_c)$
1	$a+(b+c)$	$\lvert p_a\rangle\lvert n_{bc}P_{bc}\rangle$	$E_a+E_{nP_{bc}}$	$f_aF_{n_{bc}}$	$(1-f_a)(1+F_{n_{bc}})$
2	$b+(a+c)$	$\lvert p_b\rangle\lvert n_{ac}P_{ac}\rangle$	$E_b+E_{nP_{ac}}$	$f_bF_{n_{ac}}$	$(1-f_b)(1+F_{n_{ac}})$
3	$c+(a+b)$	$\lvert p_c\rangle\lvert n_{ab}P_{ab}\rangle$	$E_c+E_{nP_{ab}}$	$f_cF_{n_{ab}}$	$(1-f_c)(1+F_{n_{ab}})$

$$I_{ab}(p_a,r,t)=\frac{1}{\hbar V}\int\frac{d^3p_b}{(2\pi\hbar)^3}\frac{d^3\bar{p}_a}{(2\pi\hbar)^3}\frac{d^3\bar{p}_b}{(2\pi\hbar)^3}|\langle p_ap_b|T_{ab}|\bar{p}_a\bar{p}_b\rangle|^2 2\pi\delta(E_{ab}-\bar{E}_{ab})$$
$$\times\{\bar{f}_a\bar{f}_b(1-f_a)(1-f_b)-(1-\bar{f}_a)(1-\bar{f}_b)f_af_b\}. \tag{2.7}$$

The three-particle collision integral reads

$$I_{abc}(p_a,r,t)=\frac{1}{2}\frac{1}{\hbar V}\sum_\kappa\int\frac{d^3p_b}{(2\pi\hbar)^3}\frac{d^3p_c}{(2\pi\hbar)^3}d(\kappa\bar{\alpha})|\langle p_ap_bp_c|T^{0\kappa}_{abc}|\kappa\bar{\alpha}\rangle|^2 2\pi\delta(E_0-\bar{E}_\kappa)\{\bar{f}_\kappa N_0-\bar{N}_\kappa f_af_bf_c\}$$
$$+\frac{1}{2\hbar V}\sum_{n_{bc}}\sum_\kappa\int\frac{d^3P_{bc}}{(2\pi\hbar)^3}d(\kappa\bar{\alpha})|\langle p_aP_{bc}n_{bc}|T^{1k}_{abc}|\kappa\bar{\alpha}\rangle|^2 2\pi\delta\left(E_1-\bar{E}_\kappa\right)\left(\bar{f}_\kappa N_1-\bar{N}_\kappa f_aF_{n_{bc}}\right). \tag{2.8}$$

The collision term (2.8) contains all the possible three-body scattering processes with free and bound particles. In order to classify the different processes, we have applied the notation of multichannel scattering theory explained in Table I.

The last contribution on the right-hand side of Eq. (2.1) follows from the first-order gradient expansion of the kinetic equation. It ensures energy conservation in binary collision approximation and the compensation of the secular divergencies in I_{abc}.

As can be seen from (2.8), the solution of the kinetic equation (2.1) requires the knowledge of the distribution function of the bound particles. The kinetic equation of the bound states can be derived very similarly to that of the free particles. The explicit expression of the corresponding three-particle collision integral can be found in [16, 20].

At this point we have to remark that the collision integrals for plasmas in strong electric fields are modified by the field which results in an explicit field dependence. But these modifications are unknown up to now for the level of approximation used in this paper. The field dependence of the two-particle collision integral is known in first Born approximation. One gets [21–25]

$$I_{ab}=\frac{2}{\hbar^2}\int\frac{d\bar{k}'_a d\bar{k}_b dk'_b}{(2\pi\hbar)^9}\delta\left(k_a+k'_b-\bar{k}_a-\bar{k}'_b\right)V_s^2(k_a-\bar{k}_a)$$
$$\times\int_0^\infty d\tau\,2\cos\left\{\frac{1}{\hbar}\left[(\epsilon_a+\epsilon'_b-\bar{\epsilon}_a-\bar{\epsilon}'_b)\tau-\frac{E\tau^2}{2}\left(\frac{e_ak_a}{m_a}+\frac{e_bk'_b}{m_b}-\frac{e_a\bar{k}_a}{m_a}-\frac{e_b\bar{k}'_b}{m_b}\right)\right]\right\}$$
$$\times\left\{f_af'_b(1-\bar{f}_a)(1-\bar{f}'_b)-\bar{f}_a\bar{f}'_b(1-f_a)(1-f'_b)\right\}, \tag{2.9}$$

where $f_a=f(k_a-eE\tau,T-\tau)$. This field-dependent collision integral contains two important effects: (i) the collisional broadening, which is a memory effect also existing in the zero-field case; and (ii) the intracollisional field effect, which is determined by the field-dependent two-particle dynamics. In the following we will consider the collision integrals in the form given by (2.7) and (2.8) and account for the field dependence of the collision integral in a simplified manner introducing a relaxation field which follows from (2.9) [26, 21, 27, 25].

III. THE ELECTRON DISTRIBUTION FUNCTION

A. Diffusion approximation

In the following a spatially homogeneous hydrogen plasma is investigated in a constant electric field with arbitrary field strength E. In order to solve the field-dependent kinetic equation (2.1), several methods are presented in literature [12]. We will restrict us to the Fokker-Planck approximation of collision integrals. Furthermore, we account for anisotropy in a first approximation:

$$f_e=f_e^0+f_e^1\cos\vartheta, \tag{3.1}$$

where f_e^0 is the isotropic part of the electron distribution function which includes the field dependence explicitly. This is an extension of the usually used Chapman-Enskog method [28, 29]. If we insert (3.1) in the electron kinetic equation assuming the diffusion approximation in the stationary case we arrive at [13, 30]

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left\{p^2\left(\frac{eE}{3}f_e^1-\frac{m_e}{m_H}\frac{1}{\tau_e}\left[pf_e^0+mkT\frac{\partial}{\partial p}f_e^0\right]\right)\right\}=0, \tag{3.2}$$

$$eE\frac{\partial}{\partial p}f_e^0+\frac{1}{\tau_p}f_e^1=0. \tag{3.3}$$

Here the contributions of collisions are included in the energy- and momentum-relaxation time

$$\tau_e^{-1}=\left(\nu_a+\nu_p+\nu_{ao}+\frac{m_H}{m_e}\nu_e\right),\quad \tau_p^{-1}=(\nu_a+\nu_p+\nu_{a1}) \tag{3.4}$$

where the collision frequencies ν_j are connected with the quantum mechanical transport cross sections $\sigma_j^T=\int(1-\cos\theta)d\sigma$ by

$$\nu_j=\frac{p}{m_j}n_j\sigma_j^T. \tag{3.5}$$

Here $j = a$ describes the electron-atom scattering, $j = p$ the electron-proton scattering, $j = e$ the electron-electron scattering, and $j = a_0, a_1$ the excitation processes. The influence of ionization processes are described by $j = i$. Details of the derivation can be found in [13, 30].

The solution of the homogeneous equations (3.2) and (3.3) is the well-known Davydov expression

$$f_e^{0h} = C \exp\left(- \int_0^\epsilon \frac{d\varepsilon}{kT + \frac{m_H e^2 E^2 \tau_p \tau_e}{3m_e^2}} \right). \tag{3.6}$$

Some interesting special cases follow from (3.6). Assuming that the energy dependence of the relaxation times can be represented by an effective scattering cross section

$$\tau_p \tau_e = \frac{m}{2\epsilon} [n_e \sigma^{\text{eff}}(\epsilon)]^{-2}$$

we get the following limiting behavior. As a first special case we assume an energy dependence

$$\sigma^{\text{eff}}(\epsilon) = \frac{b}{\sqrt{\epsilon}}.$$

Then it follows a Maxwell distribution with an electron temperature which goes with the field squared

$$f_o(\epsilon) = c e^{-\epsilon / kT_e} \quad \text{with} \quad T_e = T_i + \frac{e^2 E^2}{6 n_e^2 b^2}.$$

If the cross section is given by a constant value the well-known Druyvesteyn distribution function for high fields [31] can be obtained

$$\sigma^{\text{eff}}(\epsilon) = b, \quad f_0(\epsilon) = c \left(\frac{\epsilon}{a} + 1 \right)^{a/kT} e^{-\epsilon/kT}$$

$$\text{with} \quad a = \frac{e^2 E^2}{6 n_e^2 b^2 kT}.$$

The limiting case

$$\sigma^{\text{eff}}(\epsilon) = b\epsilon^{-1}, \quad f_0(\epsilon) = c(1 + bE)^{-b/kT}$$

$$\text{with} \quad E < E_{\text{crit}} = 2\frac{n_e b}{e}$$

indicates that we have only normalizable distribution functions for fields below a critical value. For effective cross sections decreasing faster with energy than the last case we cannot find normalizable distribution functions. The interaction mechanism is not energetically effective enough to produce a stationary solution in this case.

Due to the observation of unnormalizable distribution functions we conclude that there exist electrons, which are accelerated unlimited theoretically. Following the idea of Gurevich [32] we can account for this fact by a nonvanishing flux, the divergence of which is equal to zero in Eq. (3.2). This means that we admit a constant flux of electrons reaching every upper limit in momentum space, but which is bounded by the system of course. In order to determine this flux we return to the differential equation (3.2) and use the general solution instead of the f_e^{0h} (3.6), which is the solution of the homogeneous equation (3.2). The general solution reads

$$f_e^0 = C_1 f_e^0 \left(1 + C_2 \int_0^\epsilon \frac{\tau_p \tau_e \frac{m_H}{m_e} d\varepsilon}{(2m_e\varepsilon)^{3/2} (\frac{e^2 E^2 \tau_e \tau_p m_H}{3m_e^2} + kT) f_e^0(\varepsilon)} \right). \tag{3.7}$$

For the secondary boundary condition, besides the normalization we choose that the momentum of electrons are limited somewhere by the boundaries

$$\frac{f_e^0}{f_e^{0h}} \to 0 \quad \text{for} \quad p \to p_{\max}. \tag{3.8}$$

The resulting distribution function will be shown in Sec. III C. But first we have to determine the quantum-mechanical cross sections which enter the distribution function according to (3.4) and (3.5).

B. Transport cross sections

Following (3.6) and (3.7) the electron distribution function is expressed in terms of the collision frequencies of the electron-scattering processes. That requires the determination of the transport cross sections and the composition of the plasma.

First the determination of transport cross sections is considered. In the case of the electron-electron and the electron-proton collisions the cross sections were calculated in a well-known manner from the scattering phase shifts by numerical solution of the radial Schrödinger's equation. The effective two-body interaction between the charged particles is assumed to be a statically screened Debye potential

$$V_{ab}^{\text{eff}}(q, 0, E) = \frac{V_{ab}(q)}{\epsilon(q, 0, E)}, \quad \epsilon(q, 0, E) = 1 + \frac{\kappa^2(E)}{q^2}, \tag{3.9}$$

where $V_{ab}(q) = 4\pi e^2/q^2$ is the Fourier transform of the Coulomb potential and $\epsilon(q, 0)$ is the random-phase approximation (RPA) dielectric function in static approximation. The influence of the electric field on the two-body interaction we have accounted for in the static dielectric function introducing a field-dependent inverse Debye radius [33]

$$\kappa^2(E) = \kappa_e^2(E) + \kappa_p^2, \quad \kappa_p^2 = 4\pi\beta n_p e^3.$$

For the electron contribution follows in the RPA static

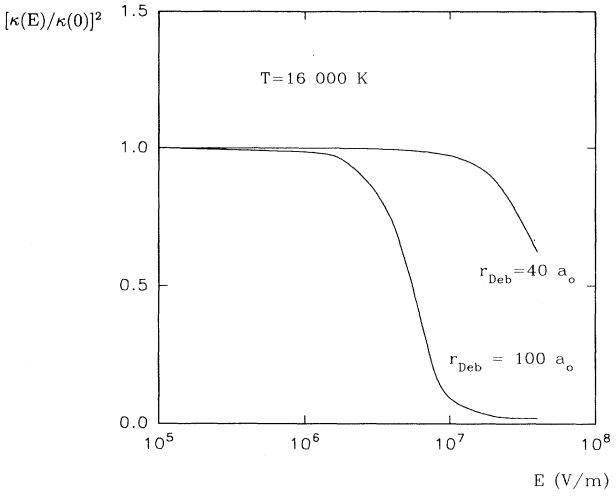


FIG. 1. The ratio of field-dependent Debye-screening parameters to field-free ones vs field strength for the two different boundary conditions and a temperature of 16 000 K and two different Debye lengths.

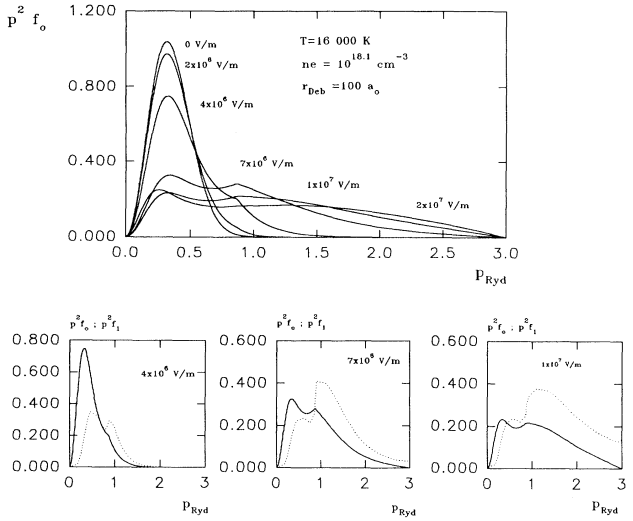


FIG. 2. Isotropic distribution function (upper) in artificial units vs momentum for a free-electron density of $10^{18.1}$, a degree of ionization of 0.1, and a temperature of 16 000 K with six different field strengths. In the lower panels, three fields are picked out together with the corresponding anisotropic part (dotted line).

approximation

$$\kappa_e^2(E) = \kappa_e^2 \frac{mkT}{2\pi^2\hbar^3 n_e} \int_0^\infty dp f_e^0(p,E). \tag{3.10}$$

Here we have considered the isotropic part only, following from the isotropic electron distribution function f_e^0 given by (3.7). As it can be seen, the field-dependent screened potential is determined in a self-consistent manner by the field-dependent electron distribution function. In Fig. 1 the screening parameter $\kappa_e(E)$ is shown as a function of the electric-field strength using the distribution function discussed in the following in Fig. 2. As expected the electric field gives rise to a weakening of the screening caused by the electrons. It should be mentioned here again that field effects for the heavy particles were neglected.

The elastic scattering of electrons on hydrogen atoms in the ground state was treated by applying the adiabatic exchange model. Starting from the close-coupling equations the approximations involved in this model permit us to reduce the quantum-mechanical three-body problem to the electron scattering in an effective atomic potential. The radial part of the electron-scattering wave function $f_l(r)$ then satisfies the integro-differential equation [34, 35]

$$\frac{d^2}{dr^2} f_e(r) + \left[k^2 - \frac{\ell(\ell+1)}{r^2} - V_{eH}^{st}(r) - V_{eH}^{pol}(r)\right] f_\ell(r)$$

$$= \pm\phi_{1s}\left\{(E_1 - k^2)\delta_{\ell,0}\int_0^\infty f_0\phi_{1s}d\bar{r} + \frac{2}{2\ell+1}\left(r^\ell\int_0^\infty f_e\phi_{1s}\bar{r}^{-(\ell+1)}d\bar{r}\right.\right.$$

$$\left.\left. + \frac{2r^{-(\ell+1)}}{(2\ell+1)}\int_0^r f_\ell\phi_{1s}\bar{r}^\ell d\bar{r} - \frac{2r^\ell}{(2\ell+1)}\int_0^r f_e\phi_{1s}\bar{r}^{-(\ell+1)}d\bar{r}\right)\right\}. \tag{3.11}$$

Here $k^2 = (2m_e)(E - E_1)/\hbar$ is the wave number and ϕ_{1s} is the ground-state wave function of a hydrogen atom. In (3.11) the upper sign refers to the singlet, the lower sign to the triplet electron-scattering state.

The static part of the atomic potential is given by

$$V_{eH}^{st}(r) = -e^2\left(\frac{1}{r} + \frac{1}{a_B}\right)e^{-2r/a_B}. \tag{3.12}$$

The polarization potential was determined in dipole approximation assuming static Debye screening in the two-body interaction potentials. Introducing a fit radius r_0 we arrive at

$$V_{eH}^{\rm pol}(r) = -\frac{e^2}{2}\frac{\alpha}{(r^2+r_0^2)^2}(1+\kappa r)^2 e^{-2\kappa r}. \tag{3.13}$$

Here α is the atomic polarizability and the parameter r_0 was chosen to be $r_0 = 1.6a_B$, which interpolates the behavior for small distances [36]. We have solved Eq. (3.11)

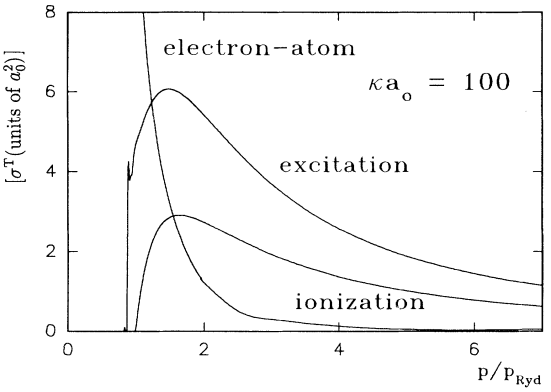


FIG. 3. Quantum-mechanical transport cross sections for Paschen excitation series in comparison with the ionization and electron-atom cross section.

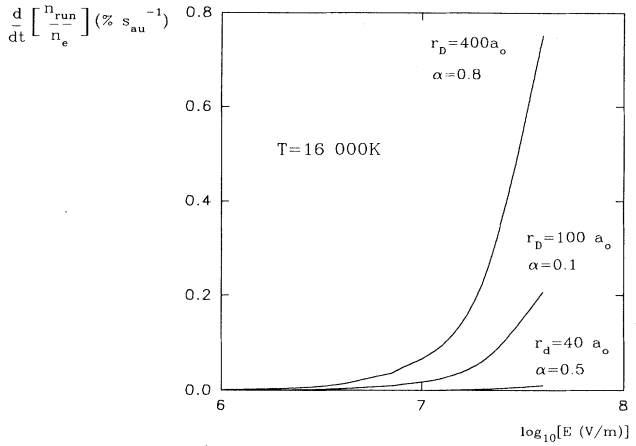


FIG. 4. The runaway electron stream (in %) of the electron density per atomic time units vs electric fields for different screening parameters in the partially ionized hydrogen plasma (see Fig. 2) and a temperature of 16 000 K.

numerically using a method given by Marriott [37] in order to determine the scattering phase shifts and finally the transport cross sections.

Additionally to the elastic electron-atom scattering we have included excitation and ionization processes from the atomic ground state by electron impact. This was done in a simplified manner using a fit formula for the total cross section given by Drawin and Emard [38]. Numerical results for the cross sections of the elastic- and the inelastic-scattering processes are shown in Fig. 3. As it can be seen, the inelastic processes are of importance at higher impact energies, which determine essentially the behavior of the electron distribution function at high electric fields.

C. Field-dependent distribution function

Using the quantum-mechanical scattering cross sections calculated above we want to discuss the main features of the field dependence of the electron distribution function (3.7) assuming a fixed plasma composition determined by a field-independent mass-action law valid for thermodynamic equilibrium. In Fig. 2 the distribution function is shown for different field strengths. As it can be seen, the electric field causes a broadening to higher energies. At energies above the first excitation threshold of electron-atom scattering the electrons cool down. This leads to a decrease of the distribution function providing stationarity. Further, it causes a second maximum at energies near 13.6 eV, which is the low-density ionization threshold. Above the critical field strength, which was here 10^7 V/m, the inelastic scattering is no longer effective enough to produce a stationary solution. The result is a nonrenormalizable distribution function, as it was discussed in Sec. III. This can be expressed by a certain runaway-electron current. There we start from the general solution of (3.2) and use the Gurevich boundary condition to fix the distribution function at a special upper limit of energy at 122.4 eV. It could be shown that the transport properties do not change if this limit is enlarged. The flux mentioned, which can be interpreted as the stream of *runaway electrons* reaching every upper limit in momentum domain [12, 30], can be seen in Fig. 4. Around the critical field strength the runaway current increases rapidly due to the long tail of the distribution function. It also indicates the breakdown of the system.

The behavior of the distribution function at higher field strengths is essentially influenced by the ionization and excitation processes. Especially, if the critical field strength, discussed above, is exceeded, stationarity will be possible only due to boundary conditions of the system. This is the reason why different attempts yield different results in this field regime. In Fig. 5 the conductivity is plotted versus the electric field for a fixed plasma composition. The result with the Davydov expression (3.6) is plotted in comparison with the Schenter-Liboff re-

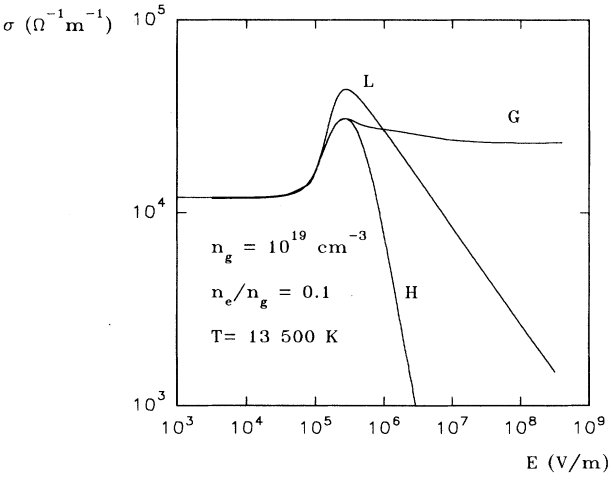


FIG. 5. Static conductivity vs electric field for different treatments. The Schenter-Liboff result (L) and the Dawydow (H) expression (3.6) together with the modified boundary solution by Gurevich (G).

sult [10]. Our used Gurevich variant (3.7) decreases the conductivity somewhat by the ionization processes and yields almost constant values at higher field strengths. This is due to the calculated runaway electron current discussed above. The difference from the Schenter-Liboff results is from the different choice of boundary conditions of the system. They used variable upper limits of the cutoff momentum. We want to stress once again that in this region no stationary solution is possible by the internal dissipation processes. This explains that different attempts yield different results in this field domain by the chosen boundaries of the system.

IV. FIELD-DEPENDENT PLASMA COMPOSITION

The second problem in calculating the electric current density is the determination at the plasma composition. According to the expression (3.6) for the electron distribution function we have to know the number densities of the free electrons, the protons, and the hydrogen atoms.

In the case of weak electric fields the linear-response theory can be applied, which allows us to determine the plasma composition from a Saha equation, valid for the thermodynamic equilibrium [3]. But this treatment is not be justified for plasmas in high electric fields. Strong deviations of the distribution function from the Maxwellian behavior caused by the electric field give rise to considerable changes in the plasma composition, which is now described by nonequilibrium rate equations instead of Saha equations.

In order to determine the composition of a hydrogen plasma in an strong electric field we start from the rate equation for the electron number density. Because we consider a nonideal hydrogen plasma this equation may be obtained from the generalized Boltzmann equation (2.1), which accounts for reaction processes and many-body effects as well as external fields. Following the papers we arrive at

$$\frac{\partial}{\partial t} n_e = \sum_{a=e,p} \sum_j \left(\alpha_a^j n_a n_H^j - \beta_a^j n_a n_e n_p \right). \tag{4.1}$$

The quantum statistical expressions for the impact ionization and three-body recombination coefficients in terms of medium-dependent scattering quantities and distribution functions can be found in [16].

In the stationary case, taking into account the atomic ground state only, we get from (4.1)

$$\frac{n_H}{n_e n_p} = \frac{\beta_e^1(E)}{\alpha_e^1(E)}, \tag{4.2}$$

where n_H is total number density of the hydrogen atom and E denotes the electric-field strength. In the zero-field case, the following relation can be found from considering the local equilibrium case and quasiparticle energies in rigid shift approximation:

$$\beta_j = \alpha_j \Lambda_e^3 \exp[-(E_j + \Delta_j - \Delta_e - \Delta_i)/k_B T]. \tag{4.3}$$

Therefore, it follows that [16]

$$\frac{n_H}{n_e n_i} = \Lambda_e^3 \sum_j \exp[-(E_j + \Delta_j - \Delta_e - \Delta_i)/k_B T]. \tag{4.4}$$

This is the well-known Saha equation for a nonideal hydrogen plasma in thermodynamic equilibrium taking into account that the averaged energy shifts are equal to the interaction parts of the chemical potentials.

Of course, the relation (4.3) and therefore the mass-action law (4.4) are not valid for plasmas which are influenced by a strong electric field. Now we have to start from the more general expression (4.2), which represents a field-dependent nonequilibrium mass-action law. It requires the calculation of the ionization as well as the recombination coefficients separately.

First, results for the coefficient of ionization from the atomic ground state in dense plasmas in a strong electric field were given in [14]. Starting from the expression

$$\alpha_e^1 = \frac{2\pi m_e}{(2\pi\hbar)^3} \int_{I_1^{\rm eff}}^{\infty} d\varepsilon f(\varepsilon) \varepsilon \sigma_1^{\rm ion}(\varepsilon), \tag{4.5}$$

which is valid for ground-state ionization and for the nondegenerate plasma state, it was calculated in a modified first Born approximation including many-body effects in the cross section and in the field-dependent effective ionization energy $I_1^{\rm eff}$

$$I_1^{\rm eff} = |E_1| + \Delta_e(E) + \Delta_p - \Delta_1. \tag{4.6}$$

In order to determine the field-dependent plasma composition from (4.2) we need the recombination coefficient too. But we will solve this problem in a first approximation introducing an effective temperature by

$$T(E) = \frac{2}{3k} \langle E_{\rm kin}(E) \rangle, \tag{4.7}$$

where the average has to be carried out with the electron distribution function given by (3.6). Then it is possible to calculate α from β using a modified equilibrium relation as given by(4.3), but now with the effective temperature (4.7). The results can be found in [14].

V. THE ELECTRICAL TRANSPORT PROPERTIES

dc conductivity

We define the electrical conductivity in a strong electrical field by the mean particle current density in the following way

$$j = \frac{e}{mn 6\pi^2 \hbar^3} \int_0^{\infty} p^3 f_1(p, E) dp = \sigma(E) E \left(1 + \frac{\delta E^{\rm rel}}{E} \right). \tag{5.1}$$

Here f_e^1 follows from (3.3) with the relaxation time τ_p and the isotropic field-dependent distribution function f_0 determined in Sec. IV. The relaxation term $\delta E^{\rm rel}$ follows from the field dependence of the collision integral (2.9) and is given by [25, 30]

$$\frac{\delta E}{E} = -\frac{\kappa_D e_a^2}{3\epsilon_o} \frac{\sum_b \frac{n_b e_b^2 (m_a + m_b) m_b}{(m_a k T_b + m_b k T_a)^2} \{1 + x^2 - (1.5 + x^2)\sqrt{\pi} e^{x^2} [1 - \Phi(x)]\} \left(1 - \frac{e_b m_a}{e_a m_b}\right)}{\sum_b \frac{e_b^2 n_b}{k T_b}}. \tag{5.2}$$

Here Φ is the standard error function [39]. The parameter x is the product $(\lambda_{ab}\kappa_D)$ of the screening parameter and the thermal de Broglie wave length [26] of *two particles*:

$$\lambda_{ab} = \hbar \left(\frac{1}{m_a} + \frac{1}{m_b}\right) \frac{1}{\sqrt{\frac{2kT_a}{m_a} + \frac{2kT_b}{m_b}}}$$

and thus describes the quantum *interference* effects.

In Fig. 6 the results are shown for the electrical conductivity as a function of the total electron density for different field strengths. The temperature is 16 000 K.

In order to demonstrate the influence of the electric field we consider the deviations of the field-dependent conductivity from the linear-response result given by the lowest curve. As expected, a strong influence of the field is observed at lower densities and it tends to higher densities as the field grows up (dotted line). This increase of the electrical conductivity at low densities can be understand easily. First, it results from the fraction of electrons with higher energies due to the longer mean free path in the lower density range.

The second important effect follows from the plasma composition results determined by the field-dependent nonequilibrium Saha equation as discussed in Sec. IV. The field dependence of the plasma composition on the conductivity can also be seen in Fig. 6. There we plotted the result of the conductivity with field-dependent mass-action law (solid line) and the result with field-independent equilibrium Saha equation (dotted line). The influence of the field on the ionization degree and therefore on the plasma composition leads to a remarkable enhancement of the nonlinear static conductivity.

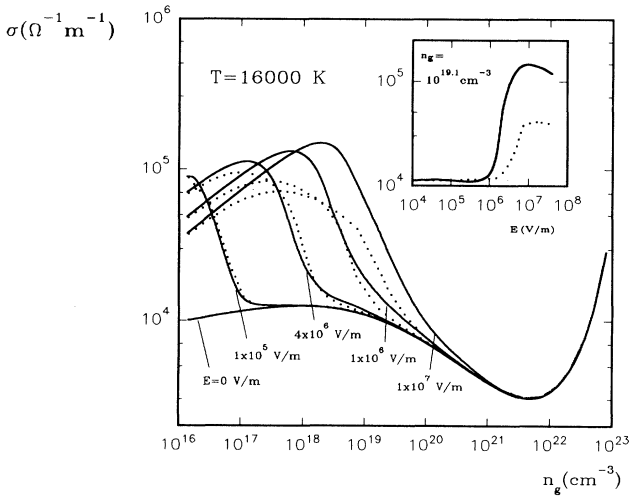


FIG. 6. The self-consistent static conductivity (solid) vs total electron density. For the reason of comparison the conductivity is plotted, where the field influence on the degree of ionization is neglected (dotted).

For lower densities the breakdown value of conductivity agrees with the non-self-consistent one. The influence of field are remarkable shifted to higher values of density in comparison to the case, where the plasma composition is not field dependent.

As our results show, it is very important to calculate the plasma composition from a field-dependent mass-action law. The higher degree of ionization due to the strong electric field gives rise to a remarkable effect in the electrical conductivity.

Of course, at higher densities the influence of the electric field on the conductivity becomes small. But now many-particle effects are of importance because of strong correlations in the plasma. We observe a minimum behavior in the electrical conductivity due to the formation of bound states. In comparison to the result (dotted curve) the minimum is reinforced because of the field effects. At densities $n_e > 10^{22}$ cm^{-3} the electrical conductivity increases drastically. This behavior, which results from the pressure ionization of hydrogen atoms in the dense plasma, describes the Mott transition in the electrical conductivity.

VI. CONCLUSION

In this paper we intend to exceed the linear transport theory to higher field strengths but remain still below the field-emission region. The field dependence of the collision integral gives rise to a renormalization field strength, which corrects the usual transport theory.

The nonlinear behavior of the conductivity is calculated. It is found that the conductivity and therefore the flux of particles is strongly enhanced in low-density regime, whereas for higher densities the many-particle effects dominate and the Mott transition occur. Between these two regions we find a window where bound states are built.

The plasma composition is determined by a nonequilibrium rate equation. The influence of the applied electric field on the ionization coefficients and therefore on the plasma composition is determined self-consistently with the found distribution function on Fokker-Planck level of approximation. A remarkable enhancement of the conductivity is found due to the field influence on the ionization coefficients.

Above a certain critical field strength the runaway electrons occur, which is connected with nonstationarity. In this regime the determination of the distribution function is crucial influenced by the chosen boundaries. This is faced by *ad hoc* assumptions about the maximum value of energy a charged particle can reach in the test system. Different attempts are compared and we have chosen the Gurevich idea to calculate the runaway electron flux explicitly.

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